Int. J. Heat Mass Transfer. Vol. 10, pp. 1291-1292. Pergamon Press Ltd. 1967. Printed in Great Britain

SHORTER COMMUNICATION

MODIFIED PLANCK MEAN COEFFICIENTS FOR OPTICALLY THIN GASEOUS RADIATION*

R. D. CESS† and P. MIGHDOLL‡
State University of New York at Stony Brook

(*Received* 14 *September* 1966 *and in revised form* 15 *April* 1967)

NOMENCLATURE

e_b, total black body emissive power [W/cm^2];
$e_{b\omega}$, spectral black body emissive power [W/cm^2/cm^{-1}];
N, number of absorption–emission bands;
P, gas pressure [atm];
$\mathbf{q}_R$, radiation heat flux vector [W/cm^2];
S_i, integrated band absorption [cm^{-2}atm^{-1}];
T, local gas temperature [°K];
T_0, arbitrary reference temperature [°K];
T_w, wall temperature [°K];
K_ω, spectral absorption coefficient [cm^{-1}];
K_m, modified Planck mean coefficient [cm^{-1}];
K_P, Planck mean coefficient [cm^{-1}];
ω, wave number [cm^{-1}].

INTRODUCTION

In dealing with radiative transfer within absorbing–emitting gases, the mathematical complexity arising from the non-gray nature of real gases has led to several definitions for an effective mean absorption coefficient. The use of such a suitable wave number average can permit the non-gray aspects of many problems to be included in a rather straightforward manner. The appropriate mean *emission* coefficient is the Planck mean coefficient. With respect to infrared gaseous radiation, results for the Planck mean coefficient have recently been presented by Abu-Romia and Tien [1] for carbon monoxide, carbon dioxide, and water vapor. The present investigation is concerned with the evaluation of corresponding mean *absorption* coefficients which apply in the limit of optically thin radiation.

Modified PLANCK Coefficients

In reference [2] it has been illustrated that when optically thin conditions prevail, the radiation transfer may be described in terms of two mean coefficients. One is the conventional Planck mean coefficient

$$K_P(T) = \frac{\int\limits_0^\infty K_\omega(T)\, e_{b\omega}(T)\, d\omega}{e_b(T)} \tag{1}$$

while the other is a modified Planck mean coefficient defined by

$$K_m(T, T_w) = \frac{\int\limits_0^\infty K_\omega(T)\, e_{b\omega}(T_w)\, d\omega}{e_b(T_w)}. \tag{2}$$

Equation (2) is actually not as general a definition as given in [2].§ Nevertheless, $K_m(T, T_w)$ as expressed by equation (2) constitutes a mean coefficient which has frequent utility. For example, if the gas is bounded by two infinite parallel black plates having the temperatures T_{w1} and T_{w2}, then from [2]

$$\operatorname{div} \mathbf{q}_R = 4K_P(T)\, e_b(T) - 2K_m(T, T_{w1})\, e_b(T_{w1}) - 2K_m(T, T_{w2})\, e_b(T_{w2}).$$

On the other hand, for an enclosure of arbitrary geometry and arbitrary surface radiation characteristics, but for which the entire bounding surface is isothermal at temperature T_w, it is easily shown that

$$\operatorname{div} \mathbf{q}_R = 4K_P(T)\, e_b(T) - 4K_m(T, T_w)\, e_b(T_w).$$

Note that unlike the Planck coefficient, the modified Planck coefficient is not an equilibrium gas property, since it is dependent upon a wall temperature. Actually, $K_P(T)$ is a mean *emission* coefficient, while $K_m(T, T_w)$, under optically thin conditions, denotes a mean *absorption* coefficient.

For infrared gaseous radiation, $e_{b\omega}$ may be assumed to

---

* This work was sponsored by the U.S. Air Force, Office of Aerospace Research, Aerospace Research Laboratories, under Contract AF 33(615)-3104.

† Professor of Engineering.

‡ Instructor of Engineering.

§ There, radiosities appeared in place of the black body emissive powers.

be independent of ω within each absorption–emission band, since the bandwidths are generally quite small. Furthermore, for each band*

$$\int_{\Delta\omega_i} K_\omega(T)\,d\omega = PS_i(T) \tag{3}$$

with $\Delta\omega_i$ denoting the width of the ith band. In addition

$$S_i(T) = \frac{T_0}{T} S_i(T_0)\,F_i(T) \tag{4}$$

where $F_i(T) = 1$ for fundamental and pure rotation bands, while $F_i(T)$ differs from unity for overtone and combination bands.

Employing equations (3) and (4), equations (1) and (2) combine to yield

$$\frac{K_m(T, T_w)\,T}{K_P(T_w)\,T_w} = \frac{\sum\limits_{i=1}^{N} e_{b\omega_i}(T_w)\,S_i(T_0)\,F_i(T)}{\sum\limits_{i=1}^{N} e_{b\omega_i}(T_w)\,S_i(T_0)\,F_i(T_w)} \tag{5}$$

where $e_{b\omega_i}$ represents $e_{b\omega}$ evaluated at the center of band i. Since $F_i(T) \neq F_i(T_w)$ only for the weaker overtone and combination bands, one might anticipate that a reasonable approximation to equation (5) is simply

$$K_m(T, T_w) = K_P(T_w)\,\frac{T_w}{T}. \tag{6}$$

The evaluation of equation (5) has been performed for carbon monoxide, carbon dioxide, and water vapor. The values of $S_i(T_0)$ and $F_i(T)$ which were employed are summarized in [2], with the exception of $F_i(T)$ for the first overtone band of carbon monoxide, which was taken from [3].

The results of these calculations are summarized in Table 1 and illustrate that equation (6) is indeed an excellent approximation to equation (5). Thus, the modified Planck mean coefficient, which is applicable under optically thin conditions, is directly related, through equation (6), to the Planck mean coefficient evaluated at temperature T_w. Values of the Planck mean coefficient for carbon monoxide, carbon dioxide, and water vapor have been reported by Abu-Romia and Tien [1,2].

Table 1. Values of $K_m(T, T_w)\,T/K_P(T_w)T_w$

Gas	T_w (°K)	$T/T_w = \frac{1}{2}$	$T/T_w = 2$
CO	400	1·000	1·000
CO	800	1·000	1·000
CO	1200	0·999	1·003
CO_2	400	1·000	1·000
CO_2	800	0·999	1·003
CO_2	1200	0·996	1·013
H_2O	400	1·000	1·000
H_2O	800	1·000	1·000
H_2O	1200	0·999	1·005

REFERENCES

1. M. M. ABU-ROMIA and C. L. TIEN, Appropriate mean absorption coefficients for infrared radiation of gases, To be published.
2. E. M. SPARROW and R. D. CESS, *Radiation Heat Transfer*. Wadsworth, Belmont (1966).
3. J. C. BREEZE and C. C. FERRISO, General Dynamics/Convair, Report No. GD/C-DBE65-007 (May 1965).

---

* Note that the spectral coefficient K_ω is the volumetric absorption coefficient; i.e. it has the dimensions of reciprocal length.